A Novel Approach to Rapid Loudspeaker Design and Prototyping

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A new approach to designing and simulating closed and vented box loudspeaker systems is presented. Unlike other software solutions such as custom-written personal calculator or computer programs, this method uses a paradigm which relieves designer and user from much of the software design and implementation hassles and also presents the data in a form much more appropriate to optimization of many separate parameters. The supporting software tool is available on most personal computers, and the technique has been implemented on over a half dozen different systems.

0. INTRODUCTION

Because of the work presented by Thiele [1] and Small [2]-[4] describing the systematic analysis of loudspeakers and enclosures, the correct design of direct-radiator loudspeaker systems has become far more a science than the personal art of a few self-appointed experts. The further efforts of Small and Margolis [5], Bullock and White [6] and others [7], have resulted in specific software methods for solving driver-enclosure designs. These and other articles have all used either general purpose computer systems programmed in languages such as BASIC or FORTRAN, or personal programmable calculators sporting their own peculiar languages. The task of designing and writing such a program, one quickly discovers, is divided such that about 20% of the effort goes to solving the actual problem at hand (designing direct-radiator loudspeaker systems) and the balance goes to hassling with the details of the programming language, user interface, operating system, and so forth. As a result, many of the published software packages are either limited in scope, difficult to use, or such a rats' nest of poor programming that they are nearly impossible to adapt to different applications or programming environments. We frequently encounter these problems in attempting to port many such programs from environments like Apple computers to DEC minicomputers. One supposes that you can't expect a good loudspeaker engineer to be a great software engineer, and vice-versa.

In attempting to come up with an easy to use software system for doing driver and enclosure prototyping, we thought of using a software tool originally intended for business use, the "spreadsheet calculator". The problems that needed to be solved by such a system were many: designing enclosures given specified drivers, designing drivers given specified enclosures and parts inventory, designing the individual components of drivers (voice coils, magnets, and so on) based on the requirements of the final system, prototyping new design ideas, and so forth. All this we wanted done by a single, integrated, simple-to-use tool. Most programs we reviewed would, at best accomplish but one of these objectives, and were not very interactive, in the sense that any major change in parameter setup (such as the some of the driver's Thiele-Small parameters) required us to re-run the program from scratch.

On the other hand, a spreadsheet calculator allows us to have all of the needed parameters in view at once, permitting us to change any combination of these parameters, and view the resulting new system almost immediately.

In this article, we will not investigate, in any great detail, the actual mathematical models involved in loudspeaker design, since they are already well established. Instead, we will explore the implementation of these algorithms on a typical spreadsheet calculator, showing the significant advantages over other methods.

1. THE SPREADSHEET PARADIGM

Spreadsheet calculators were originally designed to provide business people with a method of exploring various business scenarios. They are so named because, in layout, they look much like an accountant's spreadsheet of rows and columns of figures, all related in an organized fashion specific to the problem being described. Some examples of commercial spreadsheets included "Visicalc" and "Lotus 1-2-3"[1].

A spreadsheet calculator contains an array (usually 2 dimensional) of "cells" arranged in columns and rows. Each "cell" is addressed by its column and row number (often, columns are lettered and rows are numbered) and may contain a constant, a formula whose values can be either constants or references to the values in other cells, labels or simple strings to be printed on the screen, commands, or simply nothing. The contents of a cell are changed by moving a cursor to that cell and entering the new contents. The resultant values of cells that contain equations dependent on the changed value are then updated. Certain cells can be targeted as "goals", meaning that the spreadsheet will iterate through the cells that the goals cells are dependent on until the goals are reached (if possible), displaying the resultant model.

As an example of a simple model, let's look at the formula for the resultant resonant frequency of an inductor-capacitor tank circuit:

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{1.1}$$

A simple spreadsheet based on this equation, showing the formulas, is illustrated in figure 1.1:

	A	B	C
1	L:	**0.0**	henries
2	C:	**0.0**	farads
3	f:	1/(2*pi*sqrt(B1*B2))	hertz

Figure 1.1

In cell [B1] we enter the value of the inductor, and in [B2] the value of the capacitor. The spreadsheet will look at the formula in [B3], note the references to [B1] and [B2] and use those values to solve the equation. Cells [A1], [A2], [A3], [C1], [C2], and [C3] contain labels that make the spreadsheet easier to use and read. The actual appearance of the cells on the spreadsheet would only show the resultant values, not the equations. Figure 1.2 shows the same spreadsheet as it would appear with some reasonable values used for the calculations (in this example, as in all that follow, cells that can be modified by the user are highlighted).

	A	B	C
1	L:	**0.001**	henries
2	C:	**3.60e-3**	farads
3	f:	83.88	hertz

Figure 1.2

Often, the user has the option to specify the exact format of numerical presentation. For example, cell [B3] displays its value in fixed point notation with 2 decimal places, whereas cell [B2] is in scientific or "e" notation with 3 significant digits.

[1] A colleague has, half jokingly, refered to the tools described here as "Speaker 1-2-3".

In more complex applications, cells containing formulas can refer to other cells that themselves contain formulas, and so on describing potentially complex models. To illustrate with an example, here is a small model intended to calculate the resonant frequency of a loudspeaker, given its effective mass and compliance. In this example, the mechanical units are first converted to acoustical units, then those results are used to compute the resonant frequency. The relationships being solved here are:

$$f_S = \frac{1}{2\pi\sqrt{M_{AS}C_{AS}}} \tag{1.2}$$

where the effective acoustic mass is related to the mechanical mass by:

$$M_{AS} = \frac{M_D}{{S_D}^2} \tag{1.3}$$

the effective acoustical compliance is related to the mechanical compliance by:

$$C_{AS} = C_{MS}{S_D}^2 \tag{1.4}$$

and C_{MS} is the mechanical compliance, M_D is the mass and S_D is the effective projected area of the diaphragm. The resultant spreadsheet, with the formulas, would look like:

	A	B	C	D
1	Effective diameter	d:	**0.0**	cm
2	Effective area	Sd:	(pi*(C1/200)^2)	sq. m
3	Mechanical mass	Md:	**0.0**	kg
4	Mechanical compliance	Cms:	**0.0**	m/N
5				
6	Acoustical mass	Mas:	(C3/(C2^2))	
7	Acoustical compliance	Cas:	(C4*(C2^2))	
8				
9	Resonant frequency	Fs:	(1/(2*pi*sqrt(C6*C7)))	Hz

Figure 1.3

"Plugging in" values from a real driver for d, M_D and C_{MS} results in:

	A	B	C	D
1	Effective diameter	d:	**13.2**	cm
2	Effective area	Sd:	13.68e-3	sq. m
3	Mechanical mass	Md:	**15.3e-3**	kg
4	Mechanical compliance	Cms:	**1.52e-3**	m/N
5				
6	Acoustical mass	Mas:	81.7	kg/m^2
7	Acoustical compliance	Cas:	0.285e-6	m^3/N
8				
9	Resonant frequency	Fs:	33.0	Hz

Figure 1.4

The only limitation to the complexity of a model is the maximum size of the spreadsheet, which, even in the smallest version examined, is several thousand cells.

The basic requirements of a spreadsheet calculator for loudspeaker design use include:

- Calculations performed in floating-point arithmetic,
- Output formats in fixed-point and floating-point scientific notation,
- Availability of standard math operators (including exponentiation) and transcendental functions, such as $\log_{10}(x)$, $\sqrt{x}$, etc..

The further availability of graphing functions, customizable menus, database access and so forth, while not necessary, can only but enhance the versatility of the final tool.

2. ENCLOSURE APPLICATIONS

In many ways, the spreadsheet model for closed and vented box systems is the most impressive and useful. The model in use at Precision allows us to quickly design both types of enclosures given the Thiele-Small parameters of a driver. It becomes obvious that there are some very strict limitations to the bandwidth, power handling capacity and cabinet dimensions for direct radiator loudspeakers when all these parameters are juggled about. More than once has a customer been disappointed to learn of the physical impossibility of building a system consisting of a 6 inch woofer in a 10 liter enclosure that's 3 db down at 20 Hz with 5% efficiency capable of playing at levels exceeding 110 dB!

Let's take, as an example, a section from the closed box system design spreadsheet:

	A	B	C	D
1	Effective diameter	d:	**13.30**	cm
2	Maximum excursion	Xmax:	**0.70**	cm
3	Resonant frequency	Fs:	**33.0**	Hz
4	Equivalent volume	Vas:	**40.0**	L
5	Mechanical Q	Qms:	**2.04**	
6	Electrical Q	Qes:	**0.39**	
7	Total Q	Qts:	0.33	
8	Reference efficiency	n0:	0.36	%
9	Output level (1 watt)	SPL:	87.5	dB
10	Effective area	Sd:	138.9	sq. cm
11	Maximum displacement	Vd:	97.3	cu. cm
12				
13	Target system Q	Qtc:	**0.707**	
14	Enclosure damping		**2**	(0-5)
15	Recommended volume	Vb:	8.03	L
16	Tuning ratio	a:	4.61	
17	-3dB frequency	F3:	78.2	Hz
18	Peak response ripple	Rh:	0.00	dB
19	Maximum acoustic output	Par:	0.29	W
20	Maximum SPL		103.7	dB SPL
21	Maximum input power	Per:	82.9	W

Figure 2.1

In this example, we have entered the basic Thiele-Small parameters d, X_{MAX}, f_S, V_{AS}, Q_{MS} and Q_{ES} in cells [C1] through [C6]. The spreadsheet calculates the remainder of the driver parameters Q_{TS}, η_0, SPL, S_d and V_d automatically, as the needed parameters became available. Next, we enter the target system Q Q_{TC} and the enclosure damping coefficient (an empirically derived number describing cabinet stuffing) into cells [C13] and [C14]. The spreadsheet can then calculate the optimum box volume, the tuning ratio, the -3dB frequency, and so forth. If, in this example, the driver's Q_{TS} is higher than the target Q_{TC}, the spreadsheet reports "alignment not possible" in the recommended volume cell, and the rest of the enclosure data is left blank. Alternatively, the spreadsheet could be asked to find the lowest Q possible, or driver parameters needed for a given enclosure size and response function.

Spreadsheet models for both closed- and vented-box systems have been combined to allow comparison of two configurations using the same driver. In this version, a table comparing the frequency responses at select frequencies gives an easily visible comparison bewteen the two alignments:

	A	B	C	D	E
31		PERFORMANCE COMPARISON			
32			CLOSED	VENTED	
33	Enclosure volume	Vb:	8.03	19.86	L
34	-3dB frequency	F3:	78.2	44.9	Hz
35	Response ripple	Rh:	0.00	0.14	dB
36	Maximum output	Par:	0.29	0.12	W
37	Maximum SPL		103.7	100.1	dB SPL
38	Maximum input	Per:	82.9	35.3	W
39		FREQUENCY RESPONSE			
40			CLOSED	VENTED	
41		15.9	-27.70	-31.60	dB
42		20.0	-23.70	-24.46	dB
43		25.2	-19.72	-17.47	dB
44		31.7	-15.77	-10.75	dB
45		40.0	-11.93	-4.90	dB
46		50.4	-8.32	-1.36	dB
47		63.5	-5.18	-0.24	dB
48		80.0	-2.82	-0.05	dB
49		100.8	-1.34	-0.03	dB
50		127.0	-0.58	-0.03	dB
51		160.0	-0.24	-0.03	dB
52		201.6	-0.10	-0.02	dB

Figure 2.2

Further information can be derived, such as the parameters for equalization parameters in higher-order systems, inclusion of temperature-limited power handling (all of the above power figures are based on excursion limiting) and so on.

What we have not shown in the above examples is much of the underlying intermediate mathematics. While obviously vital to the final solutions, displaying these intermediate results would only serve to confuse the user. Items such as enclosure losses, conversion from mechanical to acoustical units, and so forth, can be done "off screen", or in "hidden cells". This also allows the user to incrementally build the application, as these intermediate results aid in debugging the model as it grows. There may, however, be a slight performance penalty in using many simple steps rather than a few complex ones.

The value of a spreadsheet implementation becomes apparent when you consider that the time it takes to update the entire spreadsheet when a parameter is changed is often less than 2 seconds, and the update can be deferred, if desired, until several parameter changes have been made, thus saving time in recalculating for each entry. Additionally, judicious setup of the model allows viewing and changing *all* the relevant parameters in a given application.

3. DRIVER DESIGN APPLICATIONS

At Precision, we receive requests to design new drivers for specific applications, and this requires the ability to specify drivers and driver components quickly and accurately to respond to the needs of a customer. For example, one application might require a driver with the same diameter, f_S and V_{AS} as a standard model, but a lower Q_{TS}. One way of accomplishing this is by changing the motor assembly. Changing the magnet structure itself is difficult because of the limited selection of magnet materials and configurations and the time and expense necessary in retooling for new front plates and pole pieces. But the voice coil itself can be changed readily. A portion of the driver and enclosure design spreadsheet allows the user to configure new voice coils, selecting amongst a wide variety of standard wire sizes and cross-sections, winding lengths, number of layers and so forth. All the resultant parameters that might affect the final performance of the system are calculated, including DC resistance, coil mass, X_{MAX}, Bl product, and so on. It is most enlightening to change the size of wire from, say 28 gauge to 30 gauge, or from copper to aluminum wire, leaving everything else constant, and watch what happens to the final system response as a result of the changes in Bl product, mass and DC resistance. Again, all of the answers are available within a few seconds of a parameter change. An extract from the voice coil spreadsheet looks like:

	A	B	C	D
78	Wire type:			
79	Copper	1		
80	Aluminum	2		
81	Phosphor/Bronze	3		
82	Silver	4		
83	Select wire type:	**1**		
84	WIRE PARAMETERS			
85	Select wire guage:		**31**	
86	Material		Copper	
87	Specific gravity		8.96	
88	Resistivity		1.72	
89	COIL FORM PARAMETERS			
90	Coil former diameter	df:	**3.25**	cm
91	Coil former length	Lf:	**1.30**	cm
92	Number of layers		**2**	
93	MAGNET PARAMETERS			
94	Top plate thickness	Tfp:	**0.79**	cm
95	Flux density in gap	B:	**9500.0**	gauss
96	VOICE COIL DATA			
97	Number of turns	N:	111	
98	Coil outside diameter		3.34	cm
99	Total wire length	Lw:	11.28	m
100	Length of wire in gap	l:	6.86	m
101	Bl product	Bl:	7.03	N/A
102	Coil mass	Mvc:	3.24	g
103	DC resistance	Re:	6.05	Ohms

Figure 3.1

Another spreadsheet model facilitates quick measurement of Thiele-Small and dynamic parameters of drivers using the normal free-air and reference volume tests (see the example in the appendix).

4. FURTHER APPLICATIONS AND ENHANCEMENTS

We have developed standard crossover configurations, allowing near-automatic optimization of crossovers and impedance equalization networks to the specific characteristics of the driver. Part of the spreadsheet for driver and enclosure design generates the component values for the electrical equivalent circuit for the completed system, along with an accurate model for the voice coil inductance which includes the effective shunting resistance due to eddy currents in the magnet circuit. This enables us to accurately design and simulate of crossovers loaded with realistic terminal impedances.

Currently, we are also building an application that would integrate the spreadsheet models with the engineering database. This would allow us to specify a given target response (either a set of driver parameters or a completed system specification) and have the spreadsheet either recommend the closest matching standard driver, or a plausible set of off-the-shelf components along with any needed changes.

Finally, we are exploring the possibility of interfacing the spreadsheet directly with data acquisition hardware and software, allowing real-time data to be available appropriately scaled to engineering units in selectable cells within the spreadsheet. The models may then be used in conjunction with a very sophisticated driver quality control program for production use. A hardcopy of the resulting spreadsheet may be used either as a repair ticket for out-of-spec drivers or as an assurance to the customer of the driver's performance.

5. CONCLUSIONS

The spreadsheet implementation has proven itself to be a rapid, easy to use and versatile tool for modeling the performance of drivers and systems in engineering and environment. It has shown itself especially valuable in determining the feasability of otherwise marginal configurations. The ability to generate new voice coil and magnet configurations has been especially useful in designing drivers to fit very specific and well defined application areas.

In spite of the seeming lack of a language standard for spreadsheets, we have successfully implemented these models on a wide variety of implementations ***>, including Access Technology's *SUPERCOMP-20* running on a DEC PDP-11/23, Applix' *ALIS*, an integrated office-automation system on Sun Microsystems workstations running Unix, and a puplic domain spreadsheet calculator *sc*. Only minor changes were required when porting the models from one system to another.

These models suffer, like all other simulation and prototyping systems, in that the results are only as dependable as the accuracy of the model Much effort has been expended in researching the literature to verify the models, and further hard prototyping has been done to verify that the models, no matter how theoretically accurate, do have some resemblance to practical reality. All in all, though, the results are gratifying.

Probably the most important advantage is that the spreadsheet approach frees the model developer from many of the subtle annoyances of programming. Most of the work is spent developing the model from the applications standpoint. As a software engineer, it has long been my opinion that one of the worst things to have happened to engineering is the original birth and continued resuscitation of languages such as BASIC and FORTRAN. To have to learn the silly idiosyncracies of languages like these is tantamount to cruel and unusual punishment, and a spreadsheet goes a long way to eliminating these problems.

6. ACKNOWLEDGEMENTS

Many of the actual algorithms were borrowed from the work by Small and Margolis done for programmable calculators [5], but the general concepts, of course, owe their existance to Thiele, Small, and Ashley and all the rest. The actual time to develop both the spreadsheet models and this article was ruthlessly stolen from my family.

7. APPENDIX - EXAMPLES

The model presented here calculates the Thiele-Small parameters for a driver, given a simple set of measurements of the driver in free air and in a sealed test chamber. The algorithms used are from Small in [2] and [3]. The driver's DC resistance (R_E), resonant frequency (f_S) and impedance at resonance ($R_{ES} + R_E$) is measured. Using these figures, the two frequencies f_L and f_H are determined where the impedance is:

$$r_0 = \frac{R_{ES} R_E}{R_{ES} + R_E} \tag{7.1}$$

The driver's Q due to mechanical losses Q_{MS} is calculated:

$$Q_{MS} = \frac{f_S \sqrt{r_0}}{f_H - f_L} \tag{7.2}$$

along with the Q due to electrical losses Q_{ES} is:

$$Q_{ES} = \frac{Q_{MS}}{r_0 - 1} \tag{7.3}$$

and the Q due to all losses Q_{TS} is:

$$Q_{TS} = \frac{Q_{MS} Q_{ES}}{Q_{MS} + Q_{ES}} \tag{7.4}$$

A similar set of measurements is made, but with the driver mounted in a sealed box of volume V_{CT}. These measurements will yield the values for f_{CT}, Q_{MCT} and Q_{ECT}. We can derive the rest of the needed Thiele-Small and the electromechanical parameters of the driver as follows:

The equivalent volume of compliance V_{AS}:

$$V_{AS} = V_{CT}\left(\frac{f_{CT} Q_{ECT}}{f_S Q_{ES}} - 1r * ** > ight) \right. \tag{7.5}$$

the reference efficiency η_0 (for a V_{AS} stated in liters):

$$\eta_0 = 9.64 \times 10^{-10} \frac{{f_S}^3 V_{AS}}{Q_{ES}} \tag{7.6}$$

the mechanical compliance C_{MS}:

$$C_{MS} = \frac{V_{AS}}{\rho c^2 {S_D}^2} \tag{7.7}$$

where ρ is the density of air ($1.18 kg/m^3$) and c is the velocity of propogation of sound in air ($345 m/s$)

The mechanical mass M_D is derived by:

$$M_D = \frac{1}{2\pi {f_S}^2 C_{MS}} \tag{7.8}$$

the force factor, or Bl product:

$$Bl = \left(\frac{2\pi f_S R_E M_D}{Q_{TS}} \right)^{\frac{1}{2}} \tag{7.9}$$

and the mechanical resistance due to suspension losses R_{MS}:

$$R_{MS} = \frac{B^2 l^2}{R_{ES} - R_E} \tag{7.10}$$

One further calculation derives the sound pressure level for the driver at 1 meter stimulated by 1 watt in our measurement environment:

$$SPL = 112 + 10 \log_{10}(\eta_0) \tag{7.11}$$

Of course, care must be taken throughout all the calculations in the spreadsheet model to ensure consistancy of basic units.

A simple spreadsheet model implementing these algorithms might look like:

	A	B	C	D	E
4	Test volume	Vct:	**0.00**	Liters	
5					
6	Effective diam.	D:	**0.00**	cm	
7	Effective area	Sd:	3.1416*(C6/2)^2	sq. cm	
8	DC resistance	Re:	**0.00**	Ohms	
9			Free air	Chamber	
10	Resonance	f:	**0.00**	**0.00**	Hz
11	Impedance at resonance	Res+Re:	**0.00**	**0.00**	ohm
12	Frequencies where z=		sqrt(C8*C12)	sqrt(C8*D12)	Ohm
13	Low frequency	Fl:	**0.00**	**0.00**	Hz
14	High frequency	Fh:	**0.00**	**0.00**	Hz
15	Geom. mean of Fl and Fh		sqrt(C14*C15)	sqrt(D14*D15)	Hz
16		THIELE-SMALL PARAMETERS			
17	Resonant frequency	Fs:	C11	Hz	
18	Equivalent volume	Vas:	C4*(((D11*D35)/(C11*C23))-1)	Liters	
19	Mechanical Q	Qms:	C11*sqrt(C12/C8)/(C16-C15)		
20	Electrical Q	Qes:	C22/(C12/C8-1)		
21	Total Q	Qms:	(C22*C23)/(C22-C23)		
22	Reference efficiency	n0:	(9.64e-10*C20^3*C21/C23)*100	%	
23	Output (1 watt in)	SPL:	112.+10*log(C25/100)	dB SPL	
24		ELECTRO-MECHANICAL PARAMETERS			
25	Compliance	Cms:	C21/(1.18*345^2*(C7/10000)^2)	m/N	
26	Mass	Md:	1/((6.2832*C20^2)*C29)	Kg	
27	Losses	Rms:	C32^2/(C12-C8)	Kg/s	
28	Force factor	Bl:	sqrt(6.2832*C20*C8*C30/C23)	N/A	
29					
30		Qmc:	D11*sqrt(D12/C8)/(D16-D15)		
31		Qec:	D34/(D12/C8-1)		

Figure 7.1

A working example, based on the measurements of a real driver, is shown in figure 7.2:

	A	B	C	D	E
4	Test volume	Vct:	**3.78**	Liters	
5					
6	Effective diam.	D:	**13.30**	cm	
7	Effective area	Sd:	138.9	sq. cm	
8	DC resistance	Re:	**6.30**	Ohms	
9			Free air	Chamber	
10	Resonance	f:	**33.0**	**113.0**	Hz
11	Impedance at resonance	Res+Re:	**43.5**	**40.5**	Ohms
12	Frequencies where z=		16.6	15.97	Ohms
13	Low frequency	Fl:	**20.0**	**92.5**	Hz
14	High frequency	Fh:	**57.7**	**142.0**	Hz
15	Geom. mean of Fl and Fh		33.97	114.61	Hz
16	THIELE-SMALL PARAMETERS				
17	Resonant frequency	Fs:	33.0	Hz	
18	Equivalent volume	Vas:	42.7	L	
19	Mechanical Q	Qms:	2.30		
20	Electrical Q	Qes:	0.39		
21	Total Q	Qms:	0.33		
22	Reference efficiency	n0:	0.38	%	
23	Output (1 watt in)	SPL:	87.8	dB SPL	
24	ELECTRO-MECHANICAL PARAMETERS				
25	Compliance	Cms:	1.58e-3	m/N	
26	Mass	Md:	14.8e-3	Kg	
27	Losses	Rms:	1.33	Kg/s	
28	Force factor	Bl:	7.03	N/A	
29					
30		Qmc:	7.60		
31		Qec:	1.40		

Figure 7.2

A few items are notable. First, the geometric means of the low and high frequency points on the impedance curves are calculated and displayed on row 15 to provide a check for the accuracy of frequency measurements. Both are skewed slightly higher in frequency than the corresponding resonance because of the relatively greater affect of the voice coil inductance on the higher frequency impedance measurements. Secondly, rows 30 and 31 are used to calculate the mechanical and electrical Q's of the driver in the test chamber, and the resulting data is used further to calculate the drivers equivalent volume V_{AS}.

8. REFERENCES

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